Resit Exam Quantum Physics 1 – 2025/2026

Thursday, November 27, 2025, 18:30 – 20:30

Read these instructions carefully. If you do not follow them your exam might be (partially) voided.

- This exam consists of 3 questions in 3 pages and a formula sheet at the end.
- The points for each question are indicated on the left side of the page.
- You have 2 hours to complete this exam.
- **Write your name and student number on *all* answer sheets that you turn in**.
- Start answering each exercise on a new page. It is ok to use front and back.
- Clearly write the total number of answer sheets that you turn in on the first page.
- Telephones, smart devices, and other electronic devices are **NOT** allowed.
- **This is a closed book exam**. Consulting reading material is **not** allowed.

33 pts **Question 1 – The square well**

Consider a particle in the infinite square potential well, where the potential is given by:

$$V(x) = \begin{cases} 0, & \text{for } 0 < x < a \\ \infty, & \text{otherwise} \end{cases}.$$

5 pts **a)** Solve the time-independent Schrödinger equation explicitly to show that the energy eigenfunctions are given by $\psi(x) = A\sin\left(\frac{n\pi x}{a}\right)$, and the eigenenergies given by $E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$, where *n* = 1, 2, 3, … Do not worry about determining the normalization constant.

7 pts **b)** Suppose that we place a particle in this well in a superposition of states given by: $\psi = 3i\,\psi_1 + 2\,\psi_2$. What is the expectation value for energy? What are the possible values you could obtain from an energy measurement?

5 pts **c)** What is the time-dependent wavefunction for the state above being $\psi(x, t = 0)$?

9 pts **d)** Assume that we let the particle relax into the ground state (ψ_1). What is the expectation value for the energy? What is the variance of the energy (σ_H)?

7 pts **e)** With the particle still in the ground state of the potential above, we adiabatically move the right wall of the potential well further away, to *x* = *2a*. The particle does not change its wavefunction in the process. Write down the wavefunction of the particle using the new eigenstates of this system as a basis. Are all the terms in the sum non-zero?
Handy integral: $\int \sin(ax)\sin(bx)\,dx = \left(\frac{1}{a^2-b^2}\right)(b\sin(ax)\cos(bx) - a\cos(ax)\sin(bx))$

33 pts **Question 2 – Hydrogenic atom and identical particles**

The wave function of the hydrogen atom in the ground state is:

$$\psi_{1,0,0} = \sqrt{\frac{1}{\pi a_0^3}}\,e^{-r/a_0}, \text{ where } a_0 = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2} \text{ is the Bohr radius.}$$

6 pts **a)** Sketch the probability density associated with $\psi_{1,0,0}$ as a function of *r*. Where in space does it have its maximum value?

10 pts **b)** Take into consideration that the electron wave function is in a three-dimensional space around the nucleus. Calculate the most probable value of *r* in the ground state?
Hints: Note that $\psi_{1,0,0}$ is spherically symmetric and only a function of *r*. You need to describe the probability of finding the electron somewhere in the shell between *r* and *r+dr* in the three-dimensional space and find the maximum of this function. No integration is required.

10 pts **c)** Find the expectation value for the distance between the electron and the nucleus through explicit integration.
Useful integral: $\int e^{-bx}x^3dx = -\left(\frac{e^{-bx}}{b^4}\right)(b^3x^3 + 3b^2x^2 + 6bx + 6)$

8 pts **d)** Write down the ground state wavefunction of two electrons placed in the hydrogen atom. Ignore electron-electron Coulomb interaction, but do take their spins into account. What if we somehow trap "bosonic electrons" in this atom? What would change in the quantum state?

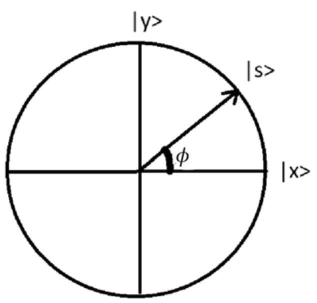
<u>34 pts</u> Question 3 – Photons as an analogue for a spin-1/2 system

For this question we will consider the polarization of photons as a quantum system and see that in the quantum world, adding additional barriers can actually increase the transmittance probability. The polarization of photons indicates the direction the electromagnetic field of the photon oscillates. This oscillation direction is perpendicular to the direction of motion of the photon. Assume the photon moves along the z-axis. The polarization can then be represented in Dirac notation as:

$$|x\rangle = \begin{pmatrix}1\\0\end{pmatrix}, \quad |y\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

or a linear combination of these states.

For this problem we will consider linear polarizations that do not change in time.



8 pts **a)** A general polarization state $|s\rangle$ can be represented in the basis of $|x\rangle$ and $|y\rangle$ as given in the image above. Give the representation for the state $|s\rangle$ as in the basis spun by $|x\rangle$ and $|y\rangle$ and show that it is normalized. For this, remember that $|s\rangle$ can be represented as a vector as shown in the figure above.

When a photon goes through a polarizing filter, the probability amplitude of it passing through it is given by the projection of the state of the photon onto the eigenstate of the filter. This is analogous to spin-1/2 particles passing through a Stern-Gerlach experiment.

The eigenstate of the polarizing filter is given by a vector along its polarization axis, e.g. for a filter along the *y*-axis, its eigenstate $|p\rangle$ is given by:

$$|p\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

and for a polarizer at 45 degrees with the *x*-axis the eigenstate is:

$$|p\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

7 pts **b)** What is the probability of an *x*-polarized photon passing through a polarizer oriented along the *y*-axis? And what if the polarizer is at 45 degrees with the x-axis? Do not simply state the probability, but show how you get to this value.

8 pts **c)** What is the probability of the *x*-polarized photon passing through two polarizers in series, one at a 45 degrees angle and then another along the *y*-direction? For this, start with calculating the probability of the photon passing through the 45 degrees polarizer, give the state of the photon after passing this 45 degrees-oriented polarizer, and then calculate the probability of it passing through the *y*-oriented polarizer.

5 pts **d)** Now consider that we have two photons with the combined quantum state:

$$\frac{1}{\sqrt{2}}(\,|x\rangle\,|y\rangle + |y\rangle\,|x\rangle\,)$$

Is this an entangled state? Why/why not?

6 pts **e)** With the combined state for two photons given in **(d)**, what is the probability of **<u>both photons</u>** passing through a *x*-polarized filter?
And what is the probability of **<u>one photon</u>** passing through an *x*-polarizer and **<u>the other photon</u>** passing through a *y*-polarizer?

Useful formulas:

Schrödinger equation: $i\hbar \frac{\partial \Psi}{\partial t} = H\Psi$

Time-independent Schrödinger equation: $H\psi = E\psi \qquad \Psi = \psi e^{-iEt/\hbar}$

Hamiltonian operator: $H = -\frac{\hbar^2}{2m}\nabla^2 + V$

Momentum operator: $p = -i\hbar\,\nabla$

De Broglie wavelength: $\lambda = h/p$

Time-dependence of expectation value: $\frac{d\langle Q\rangle}{dt} = \frac{i}{\hbar}\langle [H,Q]\rangle + \langle \frac{\partial Q}{\partial t}\rangle$

Definition of variance: $\sigma_A = \sqrt{\langle A^2\rangle - \langle A\rangle^2}$

Generalized uncertainty principle: $\sigma_A\sigma_B \geq \left|\frac{1}{2i}\langle [A,B]\rangle\right|$

Heisenberg Uncertainty principle: $\sigma_x\sigma_p \geq \hbar/2$

Canonical commutator: $[x,p] = i\hbar$

Angular momentum: $\vec{L} = \vec{r} \times \vec{p}$

$$[L_x, L_y] = i\hbar L_z \;;\; [L_y, L_z] = i\hbar L_x \;;\; [L_z, L_x] = i\hbar L_y$$

In spherical coordinates:

$$L_z = -i\hbar\, {}^{\partial}/_{\partial\phi}$$

$$L_x = -i\hbar\left(-\sin\phi\, {}^{\partial}/_{\partial\theta} - \cos\phi\cot\theta\, {}^{\partial}/_{\partial\phi}\right)$$

$$L_y = -i\hbar\left(+\cos\phi\, {}^{\partial}/_{\partial\theta} - \sin\phi\cot\theta\, {}^{\partial}/_{\partial\phi}\right)$$

Ladder operators:

$$L_\pm|l, m_l\rangle = \hbar\sqrt{l(l+1) - m_l(m_l \pm 1)}\,|l, m_l \pm 1\rangle$$
$$L_\pm \equiv L_x \pm iL_y$$

Pauli matrices: $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \;;\; \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \;;\; \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Position-space and momentum-space:

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\phi(k, t=0)e^{i(kx-\omega t)}dk$$
$$\phi(k,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\Psi(x, t=0)e^{-i(kx-\omega t)}dx$$

Trigonometric relations:

$$\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$$
$$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$$

Integrals in spherical coordinates: $\int f(r,\theta,\phi)\, r^2 \sin\theta\, dr d\theta d\phi$